

Statistical Properties of Fluctuations: A Method to Check Market Behavior

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Summary. We analyze the Bombay stock exchange (BSE) price index over the period of last 12 years. Keeping in mind the large fluctuations in last few years, we carefully find out the transient, non-statistical and locally structured variations. For that purpose, we make use of Daubechies wavelet and characterize the fractal behavior of the returns using a recently developed wavelet based fluctuation analysis method. the returns show a fat-tail distribution as also weak non-statistical behavior. We have also carried out continuous wavelet as well as Fourier power spectral analysis to characterize the periodic nature and correlation properties of the time series.

1 Introduction

Financial markets are known to show different behavior at different time scales and under different socio-economic conditions. The random behavior of fluctuations in the smaller time scales and the manifestation of structured behavior at intermediate and long time scales have been well studied [1]-[13]. Many of the stock markets have shown large scale fluctuations during the past three years. Here we concentrate on the behavior of the fluctuation of the Bombay stock exchange (BSE) high price values in daily trading. The point that makes the analysis of the BSE price index interesting is the fact that it has a significant fluctuations on a shorter time scale while growing tremendously over a longer time period. The statistical properties of the fluctuations and the behavior of the returns of such a growing market are of

^{||}This paper is dedicated to the memory of Prof. J. C. Parikh, who was one of the founding fathers of econophysics in India.

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particular interest. Wavelet transform [14]-[16] based multi-resolution analysis [17, 18] has been successfully used earlier to analyze time series from various areas [19]-[21].

In this work, we analyze the BSE high price index value using both continuous and discrete wavelet transform and multifractal detrended fluctuation analysis (MF-DFA) [22]-[32]. We use the continuous wavelet transform (CWT) to analyze the behavior of the time series at different frequencies and extract the periodic nature of the series if existent. The discrete wavelet transform based method is used to find the multifractal nature of the time series. For the purpose of comparison, the MF-DFA method is used for characterization of the time series. It has been observed in [20] that BSE returns showed a Gaussian random behavior and certain non-statistical features.

The present work is organized as follows. Section 2 contains a brief description and applications of the continuous and discrete wavelet transforms. The discrete wavelet based method [21] to analyze fluctuations is reviewed in section 3. In section 4, the data is analyzed through the wavelet based method, MF-DFA and Fourier analysis. We conclude in section 5 with results and a brief discussion.

The BSE index [33] dates from July 01, 1997 to March 31, 2009. The data spans over 2903 points and is shown in Fig.1(a). As is evident from the data, the first half does not show much activity but the second half shows significant variations. Fig.1(b) depicts the logarithmic returns calculated from (6) and Fig.1(c) depicts the shuffled returns, which reveals some differences with the returns.

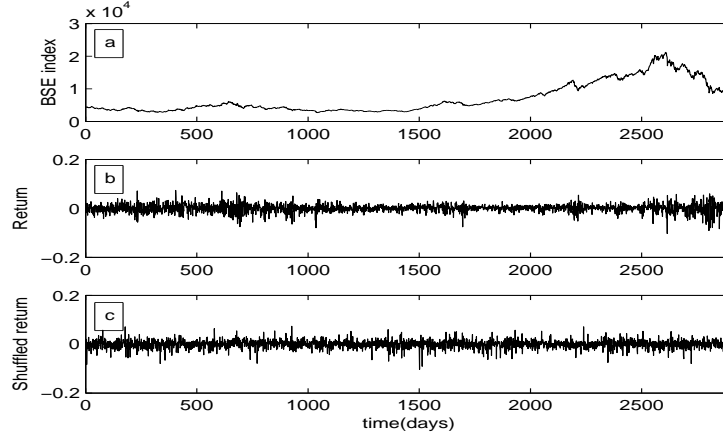


Fig. 1. (a) BSE high price index value in daily trading over a period of 2903 days. (b) Logarithmic returns estimated from Eq.6. (c) Shuffled returns.

2 Continuous wavelet analysis through Morlet wavelet

Continuous Wavelet Transform (see [34] and [41] for an excellent introduction to the topic) has been used in recent times to analyze financial time series to study self-organized criticality [36, 37], correlations [35, 38], commodity prices [39] to name a few. Recently, in [40], an

effort towards the characterization of cyclic behavior in the financial markets has been made through the multi resolution analysis of wavelet transforms. Here, the CWT of the BSE data has been carried using the Morlet wavelet given by [42],

$$\psi_0(n) = \pi^{-1/4} e^{i\omega_0 n} e^{-n^2/2} \quad (1)$$

where n is a localized time index, $\omega_0 = 6$ for zero mean and localization in both time and frequency space (admissibility conditions for a wavelet) [34]. The Morlet wavelet has a Fourier wavelength λ [41] given by

$$\lambda = \frac{4\pi s}{\omega_0 + \sqrt{2 + \omega_0^2}} \approx 1.03s \quad (2)$$

which means that here, the scale and the Fourier wavelength are approximately equal. The wavelet coefficients are calculated [41] by the convolution of a discrete sequence x_n with scaled and translated $\psi_0(n)$,

$$W_n(s) = \sum_{n'=0}^{N-1} x_{n'} \psi^* \left[\frac{(n-n')}{s} \right] \quad (3)$$

where s is the scale. The wavelet coefficients for the BSE data has been given in a scalogram in Fig.2(a) as a function of scale and time. The periodicity of the coefficients over the scales is calculated as

$$P_n = \sum_n W_n(s) \quad (4)$$

and it is given in Fig.2(b). To analyze the periodicity of the data at different frequencies,

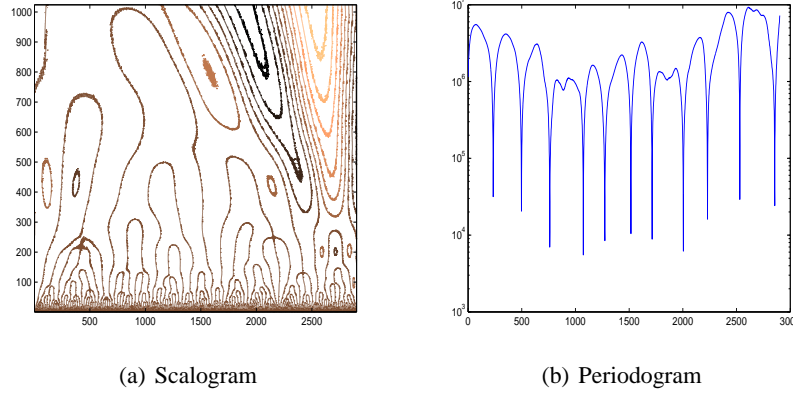


Fig. 2. (a) is the scalogram of the wavelet coefficients computed from scale 1 to 1024. The x-axis is the time, n and the y-axis is the scale s . (b) Periodogram plotted on a semilog scale as P_n vs n . One observes a period of approximately 250 trading days.

$$s \propto \nu^{-1}, \quad (5)$$

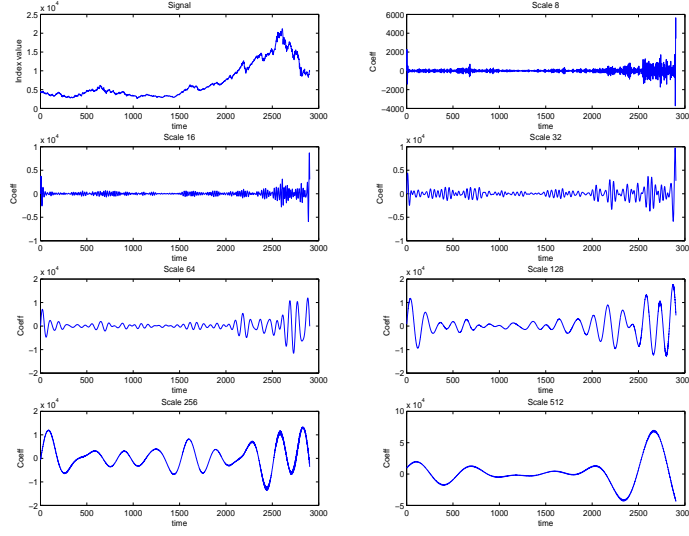


Fig. 3. Wavelet coefficients at scales 8, 16, 32, 64, 128, 256 and 512.

where v is the frequency; we have shown the $W_n(s)$ at different scales in Fig.3. One observes significant fluctuations at different scales in the second half of the data. It is evident that the fluctuations have a self similar character. We depict the fluctuations at smaller scale as also the dominant periodic variations at different scales and one does not see significant transient fluctuations in the variations.

The extracted fluctuations at different levels through DWT are shown in Fig.4(a) and Fig.4(b). Having seen the periodic behavior of the data, and having extracted the fluctuations through DWT, in the next section, we discuss the wavelet based method for analysis of fluctuations to identify their fractal behavior.

3 Discrete wavelet based method for characterizing multifractal behavior

We have observed earlier the self-similar nature of the fluctuations in the wavelet domain. In the following we describe the procedure of the wavelet based method.

From the financial (BSE stock index) time series $x(t)$, the scaled logarithmic returns $G(t)$ is defined as,

$$G(t) \equiv \frac{1}{\sigma} [\log(x(t+1)) - \log(x(t))], \quad t = 1, 2, \dots, (N-1); \quad (6)$$

here σ is the standard deviation of $x(t)$. The profile of the time series is obtained from the cumulative,

$$Y(i) = \sum_{t=1}^i [G(t)], \quad i = 1, \dots, N-1. \quad (7)$$

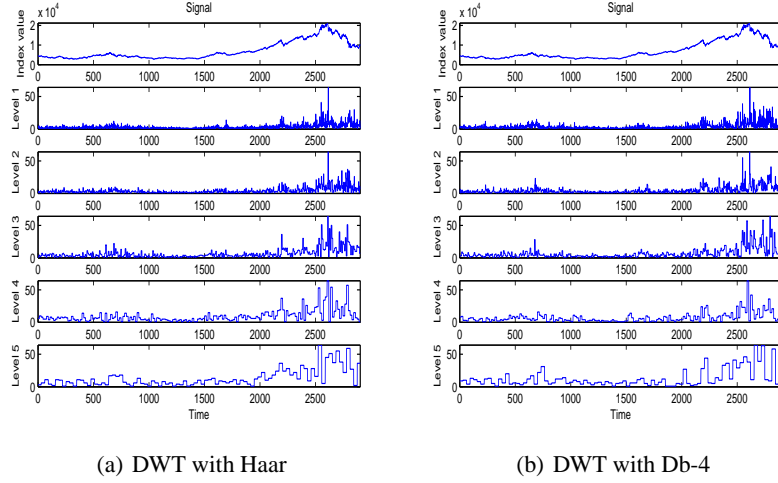


Fig. 4. (a) Discrete wavelet transform (DWT) of the data through Haar wavelet. (b) DWT of the data through Daubechies-4 (Db-4) wavelet. Akin to the CWT case, the fluctuations show self similar behavior.

Next, apply the wavelet transform on the time series profile $Y(i)$ to extract the fluctuations from the trend. The trend is extracted by discarding the high-pass coefficients and reconstructing only with low-pass coefficients using inverse wavelet transform. The fluctuations are then extracted at each level by subtracting the trend from the original time series. This procedure is followed to extract fluctuations at different levels. Here the wavelet window size at each level of decomposition is considered as the scale s . We have made use of Daubechies (Db) wavelets for the extraction of desired polynomial trend. Although the Daubechies wavelets extract the fluctuations effectively, its asymmetric nature and wrap around problem affects the precision of the values. We apply wavelet transform on the reverse profile, to extract a new set of fluctuations. These fluctuations are then reversed and averaged over the earlier obtained fluctuations.

Now the extracted fluctuations using wavelet transform are subdivided into non-overlapping segments $M_s = \text{int}(N/s)$ where N is the length of the fluctuations and s is the scale. The q^{th} order fluctuation function $F_q(s)$ is then obtained by squaring and averaging the fluctuations over all segments:

$$F_q(s) \equiv \left(\frac{1}{2M_s} \sum_{b=1}^{2M_s} [F^2(b,s)]^{q/2} \right)^{1/q}. \quad (8)$$

Here ' q ' is the order of moment. The above procedure is repeated for different scale sizes for different values of q (except $q = 0$). The power law scaling behavior is obtained from the fluctuation function,

$$F_q(s) \sim s^{h(q)}, \quad (9)$$

in a logarithmic scale for each value of q . If the order $q = 0$, direct evaluation leads to the divergence of the scaling exponent. In that case, logarithmic averaging has to be employed to find the fluctuation function:

$$F_q(s) \equiv \exp \left(\frac{1}{2M_s} \sum_{b=1}^{2M_s} \ln[F^2(b,s)]^{q/2} \right)^{1/q}. \quad (10)$$

For the monofractal time series, $h(q)$ values are independent of q and for the multi-fractal time series $h(q)$ values are dependent on q . $h(q=2) = H$, the Hurst scaling exponent is a measure of fractal nature such that varies $0 < H < 1$. Here $H < 0.5$ and $H > 0.5$ reveal the anti-persistent and persistent nature of the time series, whereas $H = 0.5$ is for random time series.

4 Data analysis and observations

The wavelet based fluctuation analysis (WBFA) which is used here was carried on the time series profile obtained from the returns and shuffled returns. The analyzed time series using discrete wavelet based method with Db-6 wavelet reveals the presence of multifractal nature with long-range correlation behavior that is shown in Fig.5(a) (top panel). For the sake of comparison, MF-DFA method with quadratic polynomial fit is also used which complements the wavelet based method (see Fig.5(a) (bottom panel)). The Hurst scaling exponent reveals that the time series possesses persistent behavior, which is shown in Table 1. The semi-log plot of distribution of logarithmic returns of BSE index and the Gaussian white noise is shown in Fig. 5(b) for the BSE index. The fat tails for large fluctuations and sharper behavior near the origin for small fluctuations are clearly seen.

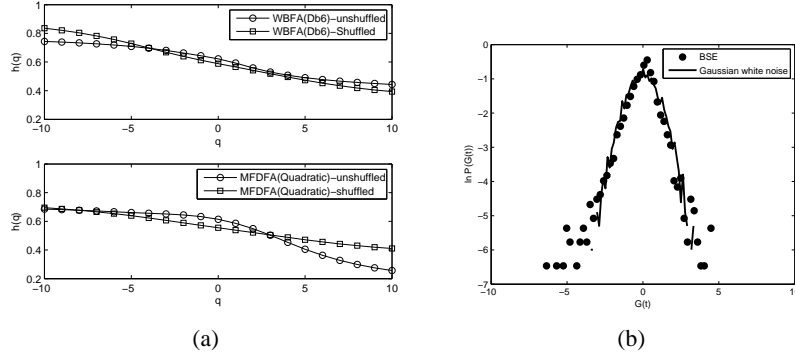


Fig. 5. (a) $h(q)$ values of BSE Sensex price index using (Top panel) WBFA (Db-6) and (Bottom panel) MF-DFA (Quadratic) analysis. (b) Log Normal Distribution of BSE sensex index return and Gaussian white noise.

X	$h(q)_{WBFA}$	$h(q)_{WBFA_s}$	$h(q)_{MFDFA}$	$h(q)_{MFDFA_s}$
Hurst Scaling Exponent	0.5486	0.5218	0.5590	0.5420

Table 1. $h(q)$ versus q values for WBFA (Db-6) and MFDFA (Quadratic) analysis.

We have also analyzed the scaling behavior through Fourier power spectral analysis,

$$P(s) = \left| \int Y(t) e^{-2\pi i s t} dt \right|^2. \quad (11)$$

Here $Y(t)$ is the accumulated fluctuations after subtracting the mean $\langle Y \rangle$. It is well known that, $P(s) \sim s^{-\alpha}$. For the BSE price index time series, the scaling exponent $\alpha = 2.11$ which reveals long range correlated behavior as shown in Fig. 6(a). The obtained scaling exponent α can be compared with Hurst exponent by the relation $\alpha = 2H + 1$. The wavelet based method and FFT are comparable.

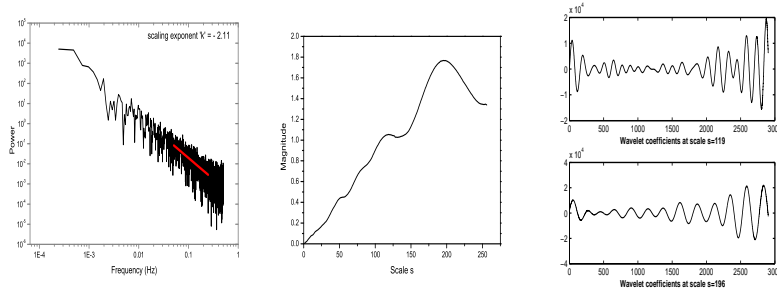


Fig. 6. (a) Fourier power spectral analysis on BSE price index. (b) Continuous wavelet analysis show two dominant periodic modulations at scale 119 and 194. (c) The CWT coefficients at the above scales (119 and 194) as a function of time.

5 Conclusion

We have analyzed BSE high price index values in daily trading. A detailed study reveals multifractal behavior and non statistical distribution of the returns. The distribution function of the returns also show fat-tail behavior. The analysis through the wavelet based method, MFDFA method and also Fourier power spectrum analysis reveal a persistent nature, as well as multifractal behavior of the BSE price index values. The multifractal nature of the time series may arise due to herding behavior, and other intrinsic non-linear character of the market and other control mechanism. We intend to study the fluctuations in different price indices in different countries for this time period. This may reveal the physical origin of the other time periods as also the multi fractal character.

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